Data Consitency Conditions of Cone Beam projections: application to the identification of acquisition parameters*

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In Cone Beam tomography, the tissue attenuation function $\mu : \mathbb{R}^3 \to \mathbb{R}$ is measured from the attenuation of X-rays emmitted from a source at $\vec{v}(t)$ acquired on a planar detector for many directions $\gamma \in \mathbb{S}^2$, where $\mathbb{S}^2$ is the unit sphere in $\mathbb{R}^3$ and $\vec{v}(t), t \in T$ is the trajectory of the source point around the patient,

$$\begin{array}{rcl} \vec{v} \ : \ T \subset \mathbb{R} & \longrightarrow & \mathbb{R}^3 \\ t & \longrightarrow & \vec{v}(t) \end{array}$$

In the tomo synthesis geometry, the source (and the detector) positions are supposed to be in a plane. The support of the attenuation function f is supposed to be in one of the half plane (separated by the source plane), between the sources and the detectors. In Fig. 1 we show an example of the circular tomosynthesis trajectory: the source path is a circle in a plane, the detector is lying in a parralel plane and the support of the measured object is between these two planes.

From now we assume that the support of f is in the half space $x_3 > 0$ and that the source positions are in the plane $x_3 = 0$, the cone beam projection from the source position $(v_1, v_2, 0) \in \mathbb{R}^3$ is defined by

$$g(v_1, v_2, \theta, \phi) = \mathcal{T} f(v_1, v_2, \theta, \phi) = \int_{\mathbb{R}^+} f\left((v_1, v_2, 0) + r\gamma_{\theta,\phi}\right) dr$$

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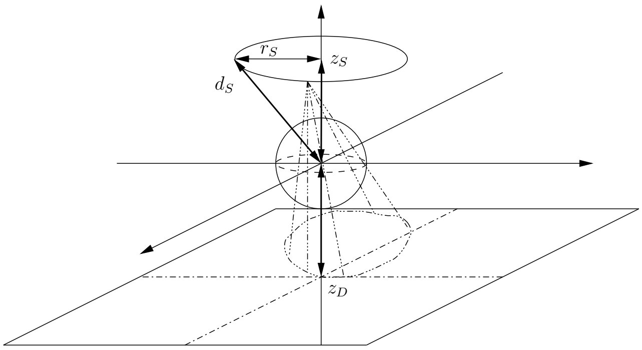

Figure 1: Circular tomosynthesis trajectory geometry.

where $\theta \in [0, \pi/2[, \phi \in [0, 2\pi[$ and $\gamma_{\theta,\phi} = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta) \in \mathbb{S}^2$ is a unit vector (θ, ϕ are its spherical angles).

Let us define

$$J_n(v_1, v_2, U, V) = \int_0^{2\pi} \int_0^{\pi/2} g(v_1, v_2, \theta, \phi)(U \cos\phi + V \sin\phi)^n \tan^n \theta \frac{\tan\theta}{\cos\theta} d\theta d\phi$$

It can be proven [3, 2] that g is in the range of $\mathcal{T}$ iff

$$J_n(v_1, v_2, U, V) = R_n(U, V, -aU - bV)$$

where $R_n(., ., .)$ is an homogeneous polynomial of degree n in its 3 variables. These new DCC (Data Consistency Conditions) can be easily computed on each projection. In many applications, DCC have been used to determine parameters of the acquisition models, see for example [6, 5, 1, 7, 8] for applications in nuclear imaging. Mathematical properties of the acquisition system can be exploited in order to design efficient calibration systems, see for exemple [4].

The main question of this master thesis is to study the possibiliy to identify information on the measurement system (such as calibration parameters) from DCC in cone beam tomography.

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