Full Cone-Beam Consistency Conditions for Sources on a Plane

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***Abstract*-- Full (necessary and sufficient) consistency conditions are presented for cone-beam projections with sources on a plane. The object support is assumed to lie entirely on one side of the source plane. We have also established full consistency conditions for planograms and linograms, in both parallel and divergent beam formats. We show that moments of the appropriately weighted cone-beam projections form polynomials in the source variables, similar to the Helgason-Ludwig conditions. The degree of the polynomial matches the degree of the moment. All the consistency conditions stated here appear to be new. A simulation example is presented for the circular tomosynthesis geometry.**

I. Introduction

In image reconstruction from projections, consistency conditions (also known as range conditions) are mathematical expressions that describe the crosstalk of information between measured projections. Consistency conditions have been widely used in reconstruction algorithms for a range of medical imaging applications, with dozens of publications over the past 20 years in SPECT (e.g. [Nat93a] [Gli94] [Men99] [Erl00]), in PET (e.g. [Def95] [Wel03] [Lay05] [Def12]), and in X-ray CT (e.g. [Bas00] [Pat02] [Hsi04] [Yu07] [Tan11]). A typical approach is to use consistency to identify the parameters of some systematic effect in the imaging model such as rigid motion parameters, a photon attenuation coefficient, a beam hardening scaling factor, an elliptical body outline, amongst various possibilities. So for many applications, a collection of necessary conditions on the projections are needed in some convenient format for processing.

Consistency conditions for parallel projections in two dimensions are well known and take different forms such as the Helgason-Ludwig (HL) conditions [Lud66] [Hel80] or the frequency-distance relation on the Fourier transform of a sinogram [Edh86]. For applications in SPECT, conditions for the exponential ray transform (also called the exponential X-ray transform) are known [Agu95], and necessary conditions are also known for the two-dimensional (2D) attenuated Radon transform [Nat83].

For cone-beam or fanbeam projections, much less is known. Of the various publications on range conditions for divergent projections (e.g. [Fin83a] [Fin83b] [Pat02] [Che05] [Yu06] [Lev10] [Cla13]) most (not all) of them are just the parallel conditions re-expressed using fanbeam or cone-beam variables. This approach presents the disadvantage that a complete set of projections must be available so that the underlying parallel geometry is completely sampled. Some of the other formulations, not related to the parallel case also require a complete set of projections [Lou89] [Nat93b] [Maz10]. For applications, it is useful to have a method of processing a small finite set of projections, so the conditions should be amenable to this preference. A discussion of this point can be found in [Cla13].

In this work we are considering cone-beam projections, and we restrict our attention to planar source trajectories with the object support being entirely on one side of the trajectory plane. Currently, consistency conditions for this geometry can be obtained by applying John's condition (see [Fin85] [Pat02]) which is a partial differential equation with the disadvantage that it only treats local information in the projections; or by considering restrictions of the cone-beam geometry to fanbeam cases [Lev10]; or by using Grangeat's result [Gra91] which also only uses lines of data on the cone-beam projections. The conditions described below treat full cone-beam projections and have the added appeal of being in the familiar form of moments of the projections, similar to the HL conditions for parallel projections. We also indicate that the consistency conditions can easily be converted to planogram coordinates [Bra04] and we give full (necessary and sufficient) consistency conditions for planogram projections in both cone-beam and parallel formats. Similar results for linograms [Edh87] are readily extracted from the existing literature and will be stated below too.

II. Theory

A. Cone-beam consistency for a planar source trajectory

Let the source trajectory be $\underline{a}(\lambda)$ and without loss of generality, define the coordinate system so that the trajectory plane is $z = 0$ and the object lies in the $z > 0$ half-space. The x and y axes can be chosen freely in the $z = 0$ plane. The unit vector $\underline{\gamma}$ is selected using conventional ϕ, θ coordinates, so $\underline{\gamma}_{\phi,\theta} = (\cos\phi\sin\theta,\ \sin\phi\sin\theta, \cos\theta)$. The cone-beam projection $g(\underline{a}, \cdot)$ is given by

$$g(\underline{a}, \underline{\gamma}_{\phi,\theta}) = \int_0^{\infty} f(\underline{a} + r\underline{\gamma}_{\phi,\theta})dr \tag{1}$$

For cone-beam consistency conditions, we first define

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$$J_n(\underline{a}, U, V) = \iint g(\underline{a}, \underline{\gamma}_{\phi,\theta})(U\cos\phi + V\sin\phi)^n \frac{\tan^{n+1}\theta}{\cos\theta} d\phi d\theta \qquad (2)$$

for each $n = 0, 1, 2\ldots$. The cone-beam projections g will then satisfy

$$J_n(\underline{a}, U, V) = R_n(U, V, -a_x U - a_y V) \qquad (3)$$

for all $\underline{a}$ in the plane, where for each n, $R_n(X, Y, Z)$ is a homogeneous polynomial of degree n.

These conditions are easily established. Simply substituting equation 1 into the expression for J_n given by equation 2, and changing from spherical to cartesian coordinates $\underline{a} + r\underline{\gamma}_{\phi,\theta} = (x, y, z)$, with $r^2 \sin\theta dr d\phi d\theta = dx dy dz$, and recalling that $a_z = 0$, we quickly obtain

$$J_n(\underline{a}, U, V) = \iiint \frac{f(x, y, z)}{z^{n+2}}(xU + yV - a_x U - a_y V)^n dx dy dz$$

$$= R_n(U, V, -a_x U - a_y V) \qquad (4)$$

where $R_n(X, Y, Z) = \sum_{i+j+k=n} c_{i,j,k} X^i Y^j Z^k$, and

$$c_{i,j,k} = \frac{n!}{i!\ j!\ k!} \iiint f(x, y, z) \frac{x^i y^j}{z^{n+2}} dx dy dz \qquad (5)$$

so R_n is a homogeneous polynomial of degree n as claimed.

These consistency conditions are *full* because the converse is also true. Given some projection function g, which satisfies equation 3 for each non-negative integer n and each $\underline{a}$ in the plane, and where R_n is a homogeneous polynomial of degree n, there exists some function f such that equation 1 is satisfied. Our proof of this fact appeals to several existing theorems in image reconstruction theory and is too long to be presented in this abstract. To the best of our knowledge, these conditions are new.

For a flat detector parallel to the trajectory plane, these consistency conditions can be written in simpler form as will be shown below.

B. Full consistency conditions for planograms

Planogram coordinates [Bra04] are a three-dimensional (3D) version of linogram coordinates [Edh85]. As shown in fig. 1, planogram coordinates are suitable for a (conceptual) PET system consisting of two parallel infinite flat detectors. An integration line is specified by absolute coordinates on one detector and relative coordinates on the second detector. For convenience it is assumed that the distance between the detectors is one (which is not restrictive because simply scaling the units will accomplish this). We will assume that the first detector is in the $z = 0$ plane and the second detector in the $z = 1$ plane. With an object f of compact support situated between the detectors we define the planogram by

$$\hat{p}(x, y, u, v) = \int f((x, y, 0) + r(u, v, 1)) dr \qquad (6)$$

Equation 6 is written $\hat{p} = \hat{P}f$ for short. The reason for the "hat" is to emphasize that the planograms are in cone-beam format. See fig. 1(a). For each source location (x, y) on the first detector, the cone-beam projection $\hat{p}(x, y, \cdot, \cdot)$ is given by equation 6. A parallel projection version is defined below.

Equations 1 and 6 are linked by associating $\underline{a} = (x, y, 0)$, and $(\cos\phi\tan\theta, \sin\phi\tan\theta, 1) = (u, v, 1)$. It is then straightforward to verify that

$$\cos\theta\ g(\underline{a}, \underline{\gamma}_{\phi,\theta}) = \hat{p}(x, y, u, v) \qquad (7)$$

and the cone-beam consistency conditions can be readily converted to full planogram consistency conditions which have a much simpler form:

P1: Let $\hat{Q}_n(x, y, U, V) = \iint \hat{p}(x, y, u, v)(uU + vV)^n du dv$. Then $\hat{p} = \hat{P}f$ for some f if and only if for all $n = 0, 1, 2\ldots$

$$\hat{Q}_n(x, y, U, V) = \hat{R}_n(U, V, -xU - yV) \qquad (8)$$

where $\hat{R}_n$ is a homogenous polynomial of degree n.

Turning now to planograms in parallel format, the variables (u, v) are the projection indices, indicating the direction of the projection, and the individual ray variables are now (x, y) (see fig. 1(b)). The idea is that a parallel projection $\bar{\bar{p}}(u, v, \cdot, \cdot)$ with direction (u, v) is measured on the first detector. We write $\bar{\bar{p}} = \bar{\bar{P}}f$ for the equation

$$\bar{\bar{p}}(u, v, x, y) = \int f((x, y, 0) + r(u, v, 1)) dr \qquad (9)$$

so $\bar{\bar{p}}(u, v, x, y) = \hat{p}(x, y, u, v)$. However, it is important to recognize that these are fundamentally different when considered as functions of their projection indices (the first 2 variables). Now theorem 4.3 in [Nat86] provides full consistency for parallel projections in three-dimensions, and a simple change of variables provides the following result:

P2: Let $\bar{\bar{Q}}_n(u, v, X, Y) = \iint \bar{\bar{p}}(u, v, x, y)(xX + yY)^n dx dy$. Then $\bar{\bar{p}} = \bar{\bar{P}}f$ for some f if and only if for all $n = 0, 1, 2\ldots$

$$\bar{\bar{Q}}_n(u, v, X, Y) = \bar{\bar{R}}_n(X, Y, -uX - vY) \qquad (10)$$

where $\bar{\bar{R}}_n$ is a homogenous polynomial of degree n.

The remarkable symmetry of the two results P1 and P2 belies their mathematical equivalence. We found that proving one from the other is not direct, and required the machinery of [Edh96]. Even for the same *f*, the polynomials $\hat{R}_n$ and $\bar{\bar{R}}_n$ are not the same in general.

C. Full consistency conditions for linograms

We present the corresponding linogram results, which are

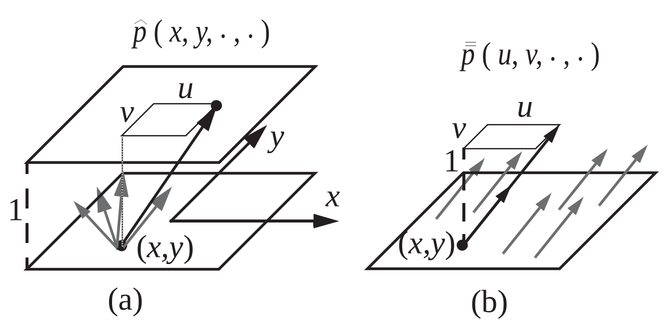

Fig 1. Planograms. (a) Cone-beam planograms. The source point (x, y) specifies the projection. The rays within a projection are specified by direction (u, v). (b) Parallel projection planograms. The direction of the projection is (u, v). The projection rays are specified by their intersection point with the detector (x, y).

the 2D versions of P1 and P2 above. Conceptually the two detector lines are vertical, separated by a distance of one, and the object lies between them.

For linograms in fanbeam format, we define $\hat{l} = \hat{L}f$ by

$$\hat{l}(y, u) = \int f((0, y) + r(1, u))dr \tag{11}$$

and an elementary substitution of variables applied to theorem 1 in [Cla13] immediately gives us

L1: Let $\hat{K}_n(y) = \int \hat{l}(y, u)u^n du$. Then $\hat{l} = \hat{L}f$ for some f if and only if for each $n = 0, 1, 2\ldots$, the function $\hat{K}_n$ is a polynomial of degree n. (I.e. $\hat{K}_n(y) = c_0 + c_1 y + \ldots c_n y^n$.)

For linograms in parallel format, we define $l = Lf$ by

$$l(u, y) = \int f((0, y) + r(1, u))dr \tag{12}$$

and we note the misleadingly simple $l(u, y) = \hat{l}(y, u)$. From the well-known 2D HL conditions, full consistency for parallel linograms are easily derived and are given by

L2: Let $K_n(u) = \int l(u, y)y^n dy$. then $l = Lf$ for some f if and only if for each $n = 0, 1, 2\ldots$, the function K_n is a polynomial of degree n. (I.e. $K_n(u) = c_0 + c_1 u + \ldots c_n u^n$.)

The symmetry in the definitions and the stated results suggest a much more trivial mathematical equivalence than actually exists between the fanbeam and parallel linograms.

III Simulations

A. Circular tomosynthesis geometry

Because recent advances have made progress with straight-line source trajectories in cone-beam tomography [Lev10] [Cla13], we chose to simulate a circular trajectory. This scanning configuration was probably first known as circular tomosynthesis. Figure 2 illustrates the geometry and an example projection of a high-contrast version of the 3D Shepp-Logan phantom. The projections were simulated using analytic line-length computations on the component ellipsoids. The detector was fixed (did not move with or conjugate to the source), and had 1024 x 1024 pixels. Thirty six projections were taken at 10^o increments along the source trajectory; this was enough projections to validate the theory and demonstrate the concepts. The cone-angle was a substantial 20^o (41^o full-angle) to ensure a strong cone-beam effect and avoid being perceived as 'nearly parallel.'

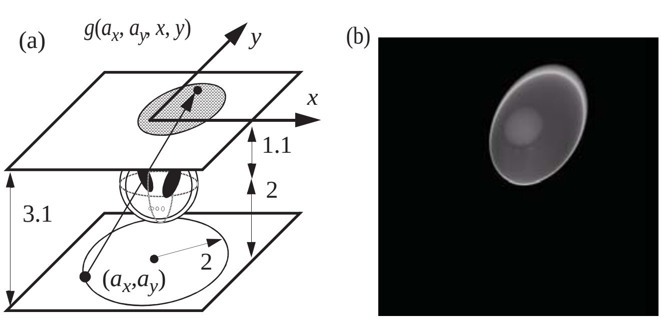

Fig 2. Circular tomosynthesis geometry. (a) the source (a_x, a_y) lies on a circle of radius 2 on the $z = 0$ plane. The center of the 3D Shepp Logan phantom is in the $z = 2$ plane, and the detector is in the $z = 3.1$ plane. Detector coordinates are (x, y). (b) One of the 36 simulated cone-beam projections.

B. Full consistency for fixed detector systems

The cone-beam measurements are given by $g(\underline{a}, \cdot)$ as specified in equation 1. We immediately scale and weight the projections so that the detector distance is one, and $\hat{g}(\underline{a}, x, y) = \cos\theta\, g(\underline{a}, \gamma_{\phi,\theta})$ with the variables linked as follows: $x - a_x = \cos\phi\tan\theta$ and $y - a_y = \sin\phi\tan\theta$. The "hat" on the g is a reminder both that the detector is fixed, and more importantly, that the raw projection measurements have been multiplied by $\cos\theta$, which is the cosine of the angle of incidence of the ray (θ is the angle between the detector normal and the incoming ray). This $\cos\theta$ term often appears in reconstruction algorithms for cone-beam tomography. To compress the notation slightly, we now use (a, b) instead of (a_x, a_y). It is easily shown that

$$\hat{g}(a, b, x, y) = \int f((a, b, 0) + r(x - a, y - b, 1))dr \tag{13}$$

which is similar to the definition of $\hat{p}$ from equation 9.

Full consistency conditions for $\hat{g}(a, b, \cdot, \cdot)$ can be written in the same form as for the planograms case. Defining $\hat{J}_n(a, b, X, Y) = \iint \hat{g}(a, b, x, y)(xX + yY)^n dxdy$, it can be shown that $\hat{g}$ satisfies equation 13 for some f if and only if, for all n, the equality

$$\hat{J}_n(a, b, X, Y) = \hat{R}_n(X, Y, -aX - bY) \tag{14}$$

holds for all (a, b) for some homogeneous polynomial $\hat{R}_n$ of degree n. (This function $\hat{R}_n$ is unrelated to the planogram $\hat{R}_n$)

C. Moment conditions on the projection data

To use the consistency conditions in practice, we first define the i-j th moment of the (cosine-scaled) projection by

$$M_{ij} = \iint \hat{g}(a, b, x, y)\ x^i\ y^j\ dxdy \tag{15}$$

and we note that for any measured projection, the number M_{ij} is easy to compute (for any pair (i, j)). Note also from the definition of $\hat{J}_n$ that (dropping the (a, b))

$$\hat{J}_n(X, Y) = c_0 Y^n + c_1 XY^{n-1} + c_2 X^2 Y^{n-2} + \ldots c_n X^n$$

$$c_k = \binom{n}{k} M_{k, n-k} \tag{16}$$

Also, the function $\hat{R}_n$ is of the form

$$\hat{R}_n(X, Y, Z) = \sum_{i+j+k=n} c_{i,j,k}\, X^i\, Y^j\, Z^k \tag{17}$$

Now, setting $Z = -aX - bY$ and equating coefficients of $X^i Y^j$ in equations 16 and 17 leads to specific consistency conditions in terms of M_{ij}.

First, for $n = 0$, we immediately have $M_{00} = c_{000}$, so

$$\iint \hat{g}(a, b, x, y)\ dxdy = c_{000} \qquad (n = 0)$$

The sum of the (cosine-weighted) cone-beam projections is the same for all projections.

For $n = 1$, we obtain $M_{10} = c_{100} - c_{001}a$ and $M_{01} = c_{010} - c_{001}b$ so

$$\iint \hat{g}(a, b, x, y)\ x\ dxdy = -c_{001}a + c_{100}$$

$$\iint \hat{g}(a, b, x, y)\ y\ dxdy = -c_{001}b + c_{010} \qquad (n = 1)$$

For $n = 2$ a similar analysis yields

$$\iint \hat{g}(a, b, x, y)\ x^2 dxdy = c_{002}a^2 - c_{101}a + c_{200}$$

$$\iint \hat{g}(a, b, x, y)\ y^2 dxdy = c_{002}b^2 - c_{011}b + c_{020} \qquad (n = 2)$$

$$\iint \hat{g}(a, b, x, y)\ xy\ dxdy = c_{002}ab - \frac{c_{011}}{2}a - \frac{c_{101}}{2}b + \frac{c_{110}}{2}$$

In summary

- M_{ij} is a polynomial of degree $i + j$ in (a, b)
- M_{n0} is a polynomial of degree n in a (resp. M_{0n} in b)
- the coefficients for each $\hat{R}_n$ can be found by fitting polynomials of degree n in (a, b) to each M_{ij} with $i + j = n$.
- for each n, the coefficients of the polynomials in (a, b) intertwine

D. Simulation results

We illustrate in fig. 3 the polynomials fit from the 36 simulated projections for the cases M_{00}, M_{10}, M_{01}, M_{20}, M_{02}. As expected for noise-free data, the fits are virtually perfect (accurate to under 0.1%), and verify the consistency theory.

V. Discussion and Conclusions

We have derived new cone-beam consistency conditions for sources lying on a plane. For consistent data the moments of the projections must be polynomials in the source variables, of the appropriate degree. Our simulations with ideal data were in agreement with the theory.

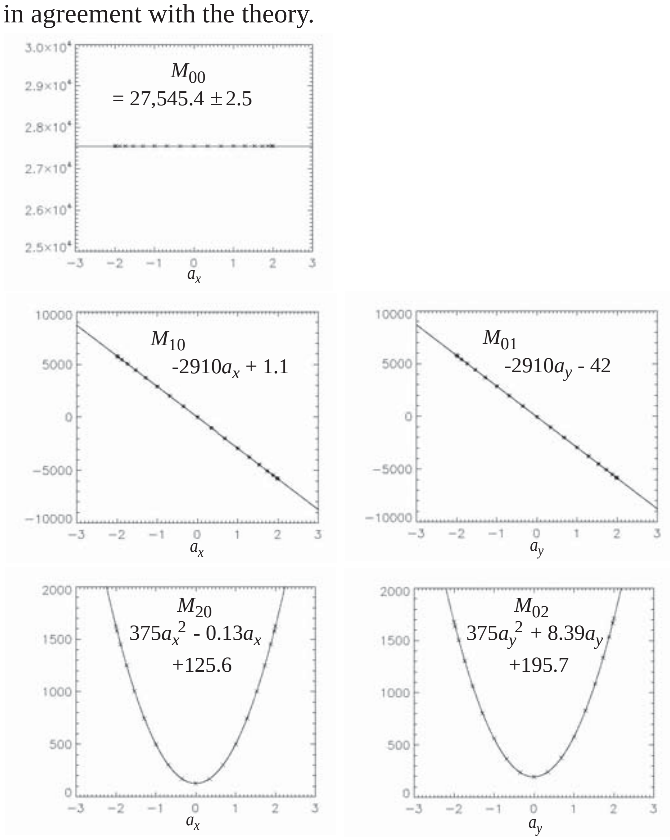

Fig 3. Projection moments, with polynomial fits. The horizontal axis is the source position. The 36 calculated values for each moment are plotted with stars (∗) and the least-squares fitted polynomial is drawn with a solid line. The results match the theory. Note that M_{10} and M_{01} have the same slope, as predicted.

The presented consistency conditions are necessary and sufficient so in principle, other conditions can be derived from them. However we have not yet linked our work mathematically to the conditions of John or Grangeat.

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